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Journal of Corporate Finance 13 (2007) 94 – 111
Journal of CORPORATE FINANCE
www.elsevier.com/locate/jcorpfin

Valuation of defaultable bonds and debt restructuring

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Received 2 June 2006; received in revised form 3 July 2006; accepted 8 July 2006
Available online 8 September 2006

Abstract

In this paper we develop a contingent valuation model for zero-coupon bonds with default. In order to emphasize the role of maturity time and place of the lender's claim in a firm's debt hierarchy, we consider a firm that issues two bonds with different maturities and different seniorage. The model allows us to analyze the implications of both debt renegotiation and capital structure of a firm on the prices of bonds. We obtain that renegotiation brings about a significant change in the bond prices and that the effect is dispersed through various channels: increasing the value of the firm, reallocating payments, and avoiding costly liquidation. Moreover, the presence of two creditors leads to qualitatively different implications for pricing, while emphasizing the importance of bond covenants and renegotiation of the entire debt.

JEL classification: G13; G32; G33
Keywords: Debt valuation; Defaultable bonds; Strategic contingent claim analysis; Modigliani–Miller theorem

1. Introduction

In recent years the work of Black and Scholes (1973) and Merton (1974) on option pricing has become an important tool in the valuation of corporate debt. The option-pricing approach has been used extensively in the valuation of stocks, bonds, convertible bonds and warrants. The theoretical insights of this approach are extremely useful, but unfortunately, the predictive power of this model has been widely challenged by empirical tests. These empirical results signaled possible limitations of the model. Two of the most important limitations are the fact that default is assumed to occur only when the firm exhausts its assets and that the firm is assumed to have a simple capital structure.

The assumption of default occurring when the firm exhausts its assets was widely criticized. These criticisms lead to the conclusion that a credit valuation model has to provide

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0929-1199/$ - see front matter
doi:10.1016/j.jcorpfin.2006.07.001

a genuine representation of the relationship between the state of the firm and the events that might influence the deterioration of the firm's value. Pursuing this goal, a new approach to credit valuation was introduced. This approach combines theory of bankruptcy and default with modern financial theory. The first to use this new approach were Leland (1994) and Leland and Toft (1996) who consider the design of optimal structure and the pricing of debt with credit risk. They allow bankruptcy to be determined endogenously and they also examine the pricing of bonds with arbitrary maturities. The problem of debt renegotiation in a contingent analysis framework was introduced by Anderson and Sundaresan (1996) who explicitly describe the interaction between bondholders and shareholders. They obtain in this way an endogenous reorganization boundary and deviations from the absolute priority rule. Mella-Barral and Perraudin (1997) also derive closed form solution for debt and equity explicitly modeling the shutdown condition for a firm while Fries et al. (1997) price corporate debt in an industry with entry and exit of firms. Allowing for contract negotiation, Mella-Barral (1999) characterizes the dynamics of debt reorganization and endogenizes departures from the absolute priority rule. In a model that allows for both value based and liquidity-based default, Fan and Sundaresan (2000) provide a framework for debt renegotiation by endogenizing both the reorganization boundary and the optimal sharing rule between equity and debt holders upon default. Finally, Anderson and Sundaresan (2000) perform a comparison among the models of Merton (1974), Leland (1994), Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), showing that the models including endogenous bankruptcy are to some extent superior to Merton's model.

A step forward in surmounting the limitation of a simple capital structure was made by Black and Cox (1976), who developed a model for pricing subordinate debt where both senior and junior debt have the same maturity. They follow Merton's approach (1974), in which risky debt is interpreted as a portfolio containing the safe assets and a short position in a put option written on the value of the firm's assets. Their junior debt could be seen as a portfolio comprising two calls: a long position in a call with a strike price equal to the face value of the senior bond and a short position in a call with a strike price equal to the sum of the face values of the two bonds.

The theory developed until now to overcome these limitations was concerned with the evaluation of credit status for securities with the same time of maturity and from the point of view of a particular lender. However, the maturity time and the place of the lender's claim in the hierarchy of the debt of a firm are also important. It is not enough that the value of the firm is sufficient for paying the debt at its maturity. Payment of later maturity debt will be affected if the firm cannot fulfill the payment obligations at interim periods. As a result, claims that have earlier maturity and are junior may trigger default and, therefore, bankruptcy.

This paper analyzes the role of multiple creditors and different capital structure on corporate debt renegotiation (when restructuring takes place through out of court renegotiation) and their effect on debt valuation. In order to do so we develop a contingent valuation model for zero-coupon bonds with different seniorities and maturities. We are interested in studying how renegotiation of debt and capital structure of the firm affect the prices of the bonds with default. Since the debt can be held by different bondholders, we permit renegotiation in case of default on the early-maturity bond and this leads to strategic behavior by bondholders. Incorporating strategic behavior by bondholders in the valuation framework suggests that the presence of renegotiation possibilities when there are multiple creditors may lead to qualitatively different implications for pricing. Our approach is similar to that taken by Anderson and Sundaresan (1996), but differs from it in two important points. First, we focus solely on the effects of bondholders' strategic behavior, the shareholders being the residual claimants in our model. Second, and more important, we consider the renegotiation of the entire amount of debt and not only of the coupon payment. This approach is used also by Christensen et al. (2002) in a single borrower setup, but the problem of renegotiating the entire amount of debt is exacerbated in our case by the strategic behavior of the two bondholders. The presence of two bondholders also helps us to emphasize the important role the bond covenants play in a firm with a richer capital structure and when we allow for the strategic behavior of bondholders. The introduction of strategic debt renegotiation prevents the liquidation and results in an increase in the value of the firm due to the elimination of the bankruptcy costs.

The remainder of this paper is organized as follows. Section 2 presents the valuation model. We directly describe the more complex model in which we allow for renegotiation. We present here the timing of the events, and the bondholders' interaction where the firm is not able to honor its payments at date 1. Section 3 studies the equilibrium of the bondholders' game. Section 4 proceeds with the valuation of the bonds. We compare the prices of the bonds in the model specified in Section 2, but also in two simpler models, the purpose of this comparison being to detect the effect on the price of bonds the capital structure of the firm and renegotiation bring about. Finally, Section 5 summarizes the results.

2. The model

There are three agents in our economy: two creditors (commercial banks, mutual or pension funds, etc.) and a firm-issuer of debt securities (corporation, commercial bank, government, etc.). All three agents are risk neutral.

The creditors live for two periods and have different liquidity preference. We assume that the preferences of the two creditors are represented by the utility function $U_i(c_1, c_2)=c_1+\delta_i c_2$, where c_1, c_2 represent the consumption of the creditors in period 1 and 2, respectively, and δ_i represents creditor i's discount factor. To emphasize the fact that the creditors have different liquidity preferences, we assume that the discount factor is very high for the first creditor, and is very small for the second one. Consequently, the first creditor prefers to consume in the first period and the second creditor prefers to consume in the second period.

Consider now a simple situation in which the current liabilities of the firm are assumed to be 0. Thus, the firm has a simple capital structure: equity and debt. Let us assume that markets are complete and frictionless, there are no taxes and the agents can borrow at the riskless interest rate r.

We assume that the firm owns a project and issues two zero coupon bonds and equity to raise funds meant to cover the financial needs of this project at date 0. As a result, the initial investment in the project is equal to the total amount raised by issuing debt and equity. There is a junior bond with face value D_1 that matures at date 1, and a senior bond with face value D_2 that is due to mature at date 2. We assume that initial value of the firm is exogenous and equal to the total investment in the project. Since our economy is characterized by 0 corporate taxes, there is no distinction between the value of the assets of the firm and the value of the firm itself. This value is $V=E+B_1+B_2$, where E is the value of equity, B_1 is the total market value of the junior corporate bond and B_2 is the total market value of the senior one. The project consists of a technology that transforms the initial investment in a random return. We model the technology as a binomial process: the value of the firm V moves up to Vu with probability p and down to Vd with probability $1-p$, where $u>1>d$. In what it follows we will denote by V_i the value of the firm at time i.

At date 0 the firm issues a short-term bond B_1 which is junior and a long-term debt B_2 which is senior.[1] There are two covenants specified in the indenture of the senior bond: limitation on priority and cross-default. The limitation on priority provision restricts the shareholders to issue additional debt that may dilute the senior bondholder's claim on the firm's assets. In our case, it requires that only junior bond can be issued in the process of debt restructuring. The cross-default provision specifies that the firm is in default when it fails to meet its obligations on any of its debt issues, that is in the case of default on the short-term debt, the senior debt becomes payable immediately.

Both bonds are subject to a positive probability of default. The existence of this positive default probability implies that the debt contracts should specify two contingency provisions: the lower reorganization boundary and the compensation to be received by the creditors when this lower reorganization boundary is reached.

The lower reorganization boundary represents the cut-off point where the liquid assets of the firm are not sufficient to meet the obligations of the debt contracts. When this cut-off point is reached, we say that financial distress takes place. As long as they meet the contractual obligations, shareholders have the residual control rights and debtholders cannot force liquidation. However, when the lower reorganization boundary is reached and shareholders default on their debt contracts, the bondholders have a choice between allowing liquidation by court appointed trustee (Chapter 7 of U.S. Bankruptcy Code) or renegotiating the debt contracts. In the case of liquidation, the firm sells its assets, pays a liquidation cost and what is left is allocated between bondholders. In the case bondholders choose to renegotiate the debt, which can be done either out of court (workout) or in court (Chapter 11 of U.S. Bankruptcy Code). Since we do not intend to model the shareholders specifically and given that the control of the firm is transferred from stockholders to bondholders in the event of default, our renegotiation procedure mirrors the restructuring through out-of-court arrangements.[2]

A very important assumption of our model is that the compensation received by bondholders after bankruptcy follows the absolute priority rule. According to this absolute priority rule, the payments to debtholders should be made before any payment is made to shareholders. Also, the payments of the debtholders are made such that the senior claim payments should always be made before any payments are made to the junior claims. We also assume that the debtholders can use the assets without any loss of value (except the liquidation costs) in the event of debt default.

2.1. Time structure

We set up the model in discrete time because it allows the modeling of the bankruptcy process to be more transparent. The sequence of events is the following:

Date 0: The firm issues both short-term and long-term debt B_1 and B_2, respectively. The promised final payments are D_1 and D_2, respectively. Creditor 1 buys the bond B_1 and Creditor 2 buys the bond B_2.

[1] The assumption is without loss of generality and is meant to illustrate the point that the junior bond with earlier maturity can trigger default on the long-term, senior bond. The case when the short-term bond is senior and the long-term bond is junior is similar to that of Black and Cox (1976) and it will not involve debt renegotiation.

[2] According to Gilson et al. (1990), almost 50% of the companies in financial distress avoid liquidation through out-of-court debt restructuring. The advantage of this procedure is that workouts are usually a lot less expensive than Chapter 11 bankruptcy procedure.

Date 1: Maturity date of bond B_1. The stockholders pay off the Bondholder 1 if they can. If they cannot, the ownership of the firm passes to the bondholders. The bondholders decide if the firm enters a liquidation or a restructuring process. In the liquidation case, the firm pays the liquidation costs L and then the bondholders are paid according to the absolute priority rule. In case of restructuring, the firm either changes the maturity of junior debt at $t=2$, or issues new debt with maturity at $t=2$. We assume that there is a cost of restructuring K and this cost is smaller than the cost of liquidation L (more precisely, we assume that $K<(r/1+r)L$, and $L<V_0d$).[3]

Date 2: Maturity date of bond B_2. Providing the firm did not go bankrupt in the previous period, the stockholders pay off the bondholders if they can. If they cannot, the firm enters liquidation. The control of the firm is transferred from stockholders to the bondholders. The firm is liquidated and the bondholders are paid according to the absolute priority rule.

2.2. Bondholders' game

At date 1, the value of the firm is V_1. The firm's payment obligation at this moment amounts to D_1. If the value of the firm V_1 exceeds D_1, the stockholders honor the debt obligation by selling assets that amount to D_1. Otherwise, the firm defaults and the stockholders give up control in favor of bondholders. Once the firm defaults on one of its payments, all the creditors have the right to demand information, and therefore they discover the value of the firm at date 1, V_1. If the value of the firm following restructuring, V_2^*, is expected to be very low (i.e. $E[V_2^*]\leq D_2$) both bondholders realize that issuing additional debt will not make them better off. Due to the existence of the senior bond covenant, the debt issued at date 1 has to be junior to the debt B_2 and therefore, the expected payment of this newly issued debt will be zero, no bondholder being willing to buy this debt. If the value of the firm is such that $E[V_2^*]>D_2$, the bondholders choose between liquidating and rescuing the firm. We consider the case when unanimity it not necessarily for the reorganization to be approved (see Franks and Torous (1989)). Consequently, liquidation occurs only when both bondholders take this decision. In the liquidation case, the assets of the firm are sold and the payments are made to the bondholders. If one of the bondholders wants to rescue the firm, then the debt will be restructured independently of the other's action. There are different ways to restructure the debt: reducing the principal obligations, increasing maturity of the debt or accepting equity of the firm. We assume that Bondholder 1 restructures the debt by increasing the maturity of the debt. On the other hand, if Bondholder 2 wants to prevent liquidation, he can do so only if the firm issues new debt. The restructuring of the debt can be done only if the firm pays a cost K, which, for simplicity, we assume to be the same in both cases.

Let us see now what happens at date 2. The situation is similar, but the payments depend on what happened at date 1. First, if the payments for the bond B_1 where made at date 1, the only payment left to be honored at date 2 is the senior bond B_2. If the value of the firm at time 2 exceeds the payment obligation D_2, the stockholders honor the debt obligation. Otherwise, they liquidate the firm and obtain the assets' value net of liquidation cost.

Second, if there is a default on the obligation at date 1, one of the three outcomes are possible: liquidation, rescue by Bondholder 1, and rescue by Bondholder 2. If liquidation takes place at date 1, the game is already over. The firm sells off the assets, pays a liquidation cost L and makes the

[3] Andrade and Kaplan (1998), Alderson and Betker (1995), Gilson et al. (1990), Gilson (1997) and Maksimovic and Phillips (1998) provide empirical evidence that the costs of out-of-court restructuring are significantly lower than the costs of liquidation.

payments according to the priority rule. Bondholder 1 owns the senior bond and he receives $\min\left\{V_1-L,\frac{D_2}{1+r}\right\}$. Bondholder 2 receives what is left, i.e. $\max\left\{V_1-\frac{D_2}{1+r}-L,0\right\}$. When restructuring takes places, the firm pays the restructuring cost K, and thus, the value of the firm becomes $V_1^*=V_1-K$. If the restructuring of the firm is made by Bondholder 1, at date 2 he is entitled to a payment D_1' which is junior to D_2. If the value of the firm at date 2, V_2^* exceeds the total payment obligation $D_1^*+D_2$, the stockholders honor the debt obligation. Otherwise, Bondholder 1 receives $\max\{0, V_2^*-D_2\}$ and Bondholder 2 receives $\min\{V_2^*, D_2\}$. If the firm is in default at date 2, we have to subtract the liquidation cost from these payoffs. Finally, if the rescue of the firm was made by Bondholder 2, at date 2 the Bondholder 2 owns two bonds and he will be entitled to a payment of $D_1''+D_2$. The payment he receives depends again on the realization of V_2^* and it is min $\{D_1^*+D_2, V_2^*\}$.

When Bondholder 2 is willing to pay the debt, the firm issues new debt which amounts to D_1. If Bondholder 2 is the only one to rescue the firm, the Bondholder 1 receives exactly the amount he received in case of liquidation $\max\left\{V_1-\frac{D_2}{1+r}-L,0\right\}$, the amount $D_1-\max\left\{V_1-\frac{D_2}{1+r}-L,0\right\}$ being used for increasing the value of the firm. Hence, the value of the firm is in this case $V_1^{**} = V_1 + D_1-K-\max\left\{V_1-\frac{D_2}{1+r}-L,0\right\}$. Finally, should both bondholders be willing to rescue the firm, the firm accepts both offers, the new value of the firm becoming in this case $V_1^{***}=V_1+D_1-2K$. The firm postpones the debt due to Bondholder 1 by changing the face value of the debt to D_1' and also by issuing new debt with face value D_1''. The two new types of debt are junior to debt B_2 and they have the same seniority.

The payments made at date 2 in the restructuring case for the new debt D_1' and D_2'' are chosen such that there are no arbitrage opportunities between the first and second periods.

3. The equilibrium of the bondholders' game

We now study the case when the firm is not able to meet its payment obligation at date 1, i.e. $V_1<D_1$, but the value of the firm is still high enough to allow for restructuring, meaning $E[V_2^*]\geq D_2$. This can be written equivalently as

$$V_1-K\geq\frac{D_2}{pu+(1-p)d}.$$

As we have already explained, the ownership of the firm passes into the hands of the bondholders and they decide whether to rescue or to liquidate the firm. We assume that the bondholders have complete information, the game is common knowledge, and that they act in their own interest. Moreover, at the beginning of the game, they can observe the realization of the firm value, V_1.

Equilibrium in the bondholders' game consists of the bondholders' actions that constitute the best response. When making the decision, the bondholders have to take into consideration both current period payoff and continuation payoff.

Proposition 1. *In the equilibrium Bondholder 1 chooses to restructure and Bondholder 2 chooses to liquidate*.

The capital structure of the firm and the covenants of the senior debt play a very important role in our model. While the cross-default provision brings about the renegotiation of the debt contracts, the limitation on priority drives the equilibrium of the bondholders' game. As we have already seen the value of the firm is greatest when both bondholders are willing to restructure the firm. The Pareto efficient equilibrium consists of bondholders restructuring and thus invigorating the firm through their action. However, in equilibrium Bondholder 2 chooses to liquidate. He does so because his overall position in the hierarchy of debt is downgraded. He held a senior bond at the outset. If both bondholders undertake restructuring Bondholder 2 will have a senior bond as before but also a junior bond. This last bond actually has the same seniority as the bond owned by Bondholder 1 and therefore the payments on these two junior bonds will be made at once. Therefore, the payments of the Bondholder 2 are reduced and in consequence he chooses to liquidate the firm.

There are also two other important issues to be taken into account when solving for the equilibrium: Bondholder 1 owns a junior debt and default occurs when the value of the firm is very small. First, Bondholder 1 owns a junior debt and he receives his payment after the senior bond payment is made. Therefore, the smaller the value of the firm, the smaller the amount that is left after the payment of the senior bond. As a result, his best response to any of Bondholder 2 actions is to restructure and increase the value of the firm. Thus, if Bondholder 2 wants to liquidate, Bondholder 1 is obviously better off by restructuring since this gives him an expected payoff that is at least as the alternative. This happens because the bondholder never undertakes restructuring when the expected payoff is smaller than the present value of the debt. If Bondholder 2 wants to rescue, Bondholder 1 is gaining even more because the value of the firm is increased more by the participation of Bondholder 2, but the newly issued debt for both bondholders has the same seniority.

Second, default occurs when the value of the firm is small. Since Bondholder 2 knows that and owns a senior bond, it does not pay for him to reinvest and accumulate debt. He prefers to leave Bondholder 1 to rescue the firm. As a result, Bondholder 2's best response to Bondholder 1's restructuring play is to liquidate. Where the value of the firm net of liquidation costs is still high enough to cover the debt due to him $D_2/1+r$, we find that the Bondholder 2's best response is to liquidate where Bondholder 1 chooses to liquidate. We also obtain that, for some small values of the parameters, the best response to liquidation is restructuring. However, for these values we have already argued that the bondholders are not going to invest and accumulate more debt because if they do, they are going to lose. Under these circumstances, we can conclude that the equilibrium of the game is: Bondholder 1 restructures and Bondholder 2 liquidates.

Corollary 2. *The equilibrium of the bondholders' game is preserved even when $K=L=0$.*

If the liquidation and restructuring costs are $K=L=0$ the equilibrium of the bondholders' game is preserved. The corollary emphasizes the fact that the equilibrium of the bondholders' game is driven by the capital structure of the firm (and the presence of covenants) and not by liquidation costs. This happens again only for the values of the parameters for which restructuring makes sense, i.e. in this case $V_1 \geq \frac{D_2}{pu+(1-p)d}$.

Once the bondholders announce their decisions, shareholders are compelled to follow the decisions of the bondholders. They play a passive role since the ownership was already been conceded to the bondholders. Since the cost associated with the restructuring process is the same, independent of who is restructuring the firm, the shareholders are indifferent between changing the maturity of bond B_1 at $t=2$ and issuing new debt.

4. The valuation of the bonds

In order to price a bond we have to compute the present value of the expected bond payments. The prices of the bonds are influenced by the characteristics of the project to be undertaken but also by the structure of the firm. We focus on determining the lower reorganization boundary and the compensation to be received by bondholders and shareholders. Once we know the payments received by every agent, we can compute the prices at date 0 for the two bonds and equity by computing the net present value of future payments. We determine the prices of the bonds in three different setups and compare the corresponding prices.

First, we compute the prices of the two bonds in the model we presented above. We want to find out how introducing debt renegotiation affects the value of the bonds. For this purpose, we will compare the prices we obtained, B_1 and B_2 with the prices of two similar bonds (the same maturity date and the same debt face value) B_1'' and B_2''. The bonds B_1'' and B_2'' are issued by a firm with a similar capital structure, but in which the bondholders are not allowed to restructure the firm in the event of default.

We will see that changes in the project characteristics (which can be seen as caused by changes in the credit quality of the issuer) induce different bond prices. However, it is not the case that only the characteristics of the project affect the valuation of the bonds. The prices of the bonds can also be influenced by the presence of other bonds with different maturity or different seniority. To isolate this effect, we compare the prices of the short-term bond B_1'' with the price of a short-term bond B_1' and the price of the long-term bond B_2'' with the price of a long-term bond B_2'. The bonds B_1' and B_2' are bonds with similar face value to B00 1 and B00 2 and each of them is a bond in a firm where this is the only debt outstanding.

Before proceeding with the valuation, let us first determine the equilibrium market interest rate. We assume that the firm that owns the project V_0 is wholly financed with equity. We determine the interest rate from the following non-arbitrage condition: an investor should be indifferent between investing in equity in the firm wholly financed by equity or in a riskless asset. On the one hand, the expected payoff from investing \$1 in equity is the total expected payment of the project divided by the value of equity q, i.e. $\frac{pV_0u+(1-p)V_0d}{q}$. Since the firm is wholly financed with equity, we obtain that $q=V_0$, so the expected payoff from investing \$1 in equity is $pu+(1-p)d$. On the other, the expected payment from investing \$1 in the riskless asset is $1+r$. As a result, our non-arbitrage condition, becomes $pu+(1-p)d=1+r$.

4.1. Bond valuation with renegotiation

As we already mentioned in the presentation of the model, the payments in the case of restructuring have to be such that there are no arbitrage opportunities. First, Bondholder 1 should be indifferent between the payment he is entitled to receive this period, D_1 and the expected payment he will obtain next period if he decides to postpone the maturity of the bond D_1' i.e. $D_1=\frac{1}{1+r}E_1[\min\{V_2^*-D_2, D_1'\}]$, with, $V_1^*=V_1-K$. Second, Bondholder 2 should be indifferent between rescuing the firm by paying D_1 at date 1 and receiving D_1^* next period. Therefore, the arbitrage condition is $D_1=\frac{1}{1+r}E_1[\min\{V_2^{**}-D_2, D_1''\}]$.

The prices of the two bonds depend critically on the relationship between the two debt face values D_1 and D_2. Note that in the event of default none of the bondholders will be willing to rescue the firm if $D_1<\frac{D_2}{pu+(1-p)d}+K\equiv\bar{V}$. In the event of default we have $V_1\leq D_1$. However, the bondholders are willing to rescue the firm only if $V_1\geq\bar{V}$. Since $V_1\leq D_1<\bar{V}$, although we allow the bondholders to renegotiate they will not be willing to restructure the firm. Hence, the value of the bonds and equity will be the same as when we do not allow for renegotiation. Therefore, the interesting case for our analysis is where $D_1\geq\bar{V}$. When looking at the effects of debt renegotiation on the prices of bonds, we will concentrate our attention only on this case because this is the one where the strategic behavior of bondholders might lead to restructuring.

However, when $\overline{V} \le D_1$ we will have both cases where bondholders are willing to restructure and where they are not. Thus, if $V_1 \le \overline{V}$, the bondholders are not willing to rescue the firm since for these values of V_1 the expected value of the firm is less than D_2, the face value of debt due to Bondholder 2 at date 2. Since the debt issued at date 1 is junior to debt D2 held by Bondholder 2, the expected payment is 0, and none of the creditors is willing to buy this debt. If $\overline{V} \le V_1 \le D_1$, the expected payment to the newly issued debt is positive and the bondholders play the game described above. The payoffs of the two bondholders depend both on the face values of the debt and on the initial value V_0.

Where $D_1 \ge \overline{V}$, the strategic behavior of the bondholders affects the payoffs of the bonds at date 1, and thus, the valuation formula is changed. In this case liquidation occurs for values of the firm smaller than a new threshold $\overline{V} = \frac{D_2}{pu+(1-p)d} + K$, this threshold being smaller than the threshold we had before (D_1). If $V_1 \le \overline{V}$, we have default; the firm liquidates its assets and the bondholders share the payments. The payoff of Bondholder 1 is $\max\left\{V_1 - L - \frac{D_2}{1+r}, 0\right\}$, while the payoff of Bondholder 2 is $\min\left\{V_1 - L, \frac{D_2}{1+r}\right\}$. If $\overline{V} \le V_1 < D_1$, the firm is not able to honor its debt obligation, but it is not liquidated. In this case, the bondholders decide to restructure the debt. In equilibrium, Bondholder 1 rescues the firm by postponing its debt maturity until date 2, the payoffs of the two bondholders being $\frac{1}{1+r} E_1\left[\max\left\{0, \min\{V_2^* - D_2, D_1'\} - LI_{\{V_1 | D_2 < V_2^* < D_2 + D_1'\}}(V_1)\right\}\right]$ for Bondholder 1 and $\frac{1}{1+r} E_1[\min\{V_2^* - L, D_2\}]$ for Bondholder 2. Where the firm does not default at date 1 the payoff of Bondholder 1 is D_1. The Bondholder 2 waits until date 2, the maturity date of its debt, and he receives the entitled debt D_2 if the value of the debt is smaller than the value of the firm. Otherwise, he receives the value of the firm net of liquidation costs.

As we can see in Fig. 1, the price of the short-term bond is increasing in V_0. There are two important kinks in the price function. The first one is the result of the upper state value of the firm becoming higher than the face value of the short-term debt D_1 (in our example when $V_0 = 6.66$) and the second where also the lower state value of the firm exceeds this amount ($V_0 = 25$).

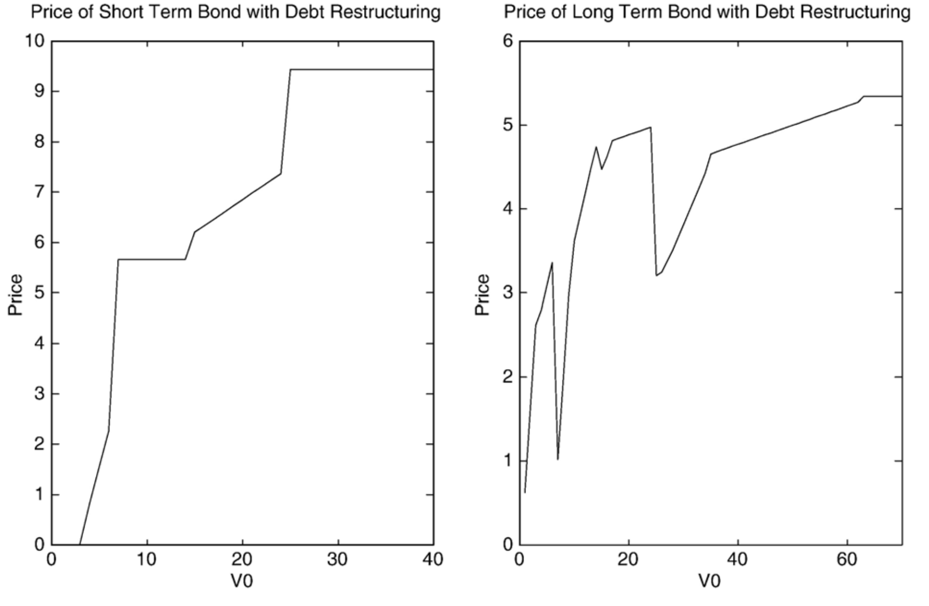

Fig. 1. Prices of the short-term and long-term bonds when debt restructuring is allowed. The values of parameters are: $D_1 = 10$, $D_2 = 6$, $K = 0.4$, $L = 0.02$, $p = 0.6$, $u = 1.5$, $d = 0.4$.

However, the price of long-term debt is no longer an increasing function of V_0. When the value of the firm becomes higher than the face value of the short-term debt D_1, the firm sells off assets amounting to D_1, and the value of the firm decreases. Consequently, for these values of V_0 we detect a sharp decrease in the price of the long-term bond.

The price of equity is computed in the same manner. In accordance with the priority rule, the equity owners are the last ones to be paid. So, if we had default at date 1 they would receive nothing. Then, if the value of parameters still allows for restructuring, we have two cases. First, if after restructuring we have default at date 2, the equity owners are left with nothing. Second, if we do not have default at date 2, they receive the value of the firm net of the payments due to the two bondholders $V_2^* - D_1' - D_2$. If we did not have default at date 1, the equity owners would receive the value of the firm minus the payment to the Bondholder 2. Once we have found the valuation formula for the two bonds and for equity, we can also compute the value of the firm V—note here that we do not obtain the initial value of the project because we have to subtract liquidation and restructuring costs. As expected, the Modigliani–Miller theorem does not hold well where such costs arise. Since we allow for renegotiation in our model, we want to see if this assumption jeopardizes the accomplishment of Modigliani–Miller theorem. Here we assume that the restructuring and liquidation costs are zero, so we can eliminate their disturbing effect. Once all the other assumptions of Modigliani–Miller theorem are fulfilled, we see what happens in our model. The first step is to determine how the behavior of the agents changes when we set $K=L=0$. As we already stated in Corollary 2, the equilibrium of bondholders game is the same when we set $K=L=0$. Since renegotiation does not involve any dissipative cost and the outcome of the project is divided between the agents, we obtain the following result:

Lemma 1. *The Modigliani–Miller theorem holds even when renegotiation is permitted if the restructuring and liquidation costs are zero*.

Hence, we find that the value of the firm remains the same even when we allow for renegotiation. The allocation of the payoffs is different when we allow renegotiation, but in the absence of liquidation and restructuring costs the value of the firm is unchanged. However, we will see later that the presence of renegotiation will offset the effect of liquidation and restructuring costs on the value of the firm when $\frac{r}{1+r}L > K > 0$.

4.2. Bond valuation with simpler capital structure

As shown above, the payments of the two bonds, and therefore, the values of the bonds depend on the face values of the debt and on the initial value V_0. Let us consider now the following two cases of a similar firm (with a similar project) but with a different capital structure. First, we consider a firm with only one outstanding bond, a bond with maturity date at $t=1$ and with face value D_1 and equity E'. Second, we consider the case of a firm with only one long-term bond outstanding with face value D_2 and equity E''.

If we assume that at date 1 the only outstanding debt is B_1', the cash flow depends only on the realization of the value of the firm V_1. If the value of the firms is high enough to pay the debt, $V_1 \geq D_1$, the bondholder receives what he is entitled to (i.e. D_1). Otherwise, he receives the amount that results from liquidating the firm. Since we assume that liquidation is costly, in the case $V_1 \leq D_1$ we have to subtract the liquidation cost L from the value of the firm. We have also computed the price of equity. As expected, the shareholders obtain nothing in case of default at date 1 and they receive $V_1 - D_1$ in case of non-default.

Now consider the second case where the firm issues a bond with maturity date 2 and with face value D_2 and it issues equity E''. When the only outstanding debt is B_2' at date 2, we are only interested if the value of the firm at $t=2$, V_2, is high enough to pay the debt. If this is the case, i.e. $V_2 \geq D_2$, the bondholder receives what he is entitled to, D_2. Otherwise, he receives the amount that results from liquidating the firm V_2-L. Similarly to the previous case, we find the price of equity. In case of default, the shareholders do not receive anything. Otherwise, they receive what is left after the payment is made to the bondholder, V_2-D_2.

4.3. Bond valuation without renegotiation

We consider now a firm with the following capital structure: equity, a zero-coupon bond B_1'' with maturity date $t=1$ and face value D_1, and a zero-coupon bond B_2'' with maturity date $t=2$ and face value D_2. However, we assume now that the bondholders are not able to rescue the firm in case of default at date 1. We obtain the prices for the two bonds B_1'' and B_2'' in a similar manner to the case when we do allow for debt restructuring.

If we compare the values of the firm we obtained in the two cases (with and without restructuring), we notice that when restructuring takes place ($\overline{V}<V_1 \leq D_1$) the value of the firm changes from V_1-L to $\frac{1}{1+r}E_1\left[V_2^*-LI_{\{V_2^*|V_2^*\leq D_2+D_1'\}}(V_2^*)\right]$. Note that if the liquidation and bankruptcy costs are different from 0, the strategic behavior induces an increase in the value of firm, and therefore, it has an offsetting effect in the violation of the Modigliani–Miller theorem.

In computing the prices, we use the interest rate to discount the payments received. To perform the comparison we need to see if the interest rate is indeed the same in all the cases. We compare the equilibrium interest rate in the case of a firm financed entirely by equity with our two cases when the firm is financed by equity, short-term debt and long-term debt and whether we allow for renegotiation. As the following lemma shows, since the bondholders and shareholders are building their expectations rationally, neither the different structure of the firm nor the presence of renegotiation changes the equilibrium interest rate.

Lemma 2. *If the firm is financed by equity, short-term and long-term debt, the interest rate satisfies* $pu+(1-p)d=1+r$ *independently of whether we allow for renegotiation or not.*

4.4. Price comparison

Let us now consider the two cases of the firm with the same capital structure: a short-term bond, a long-term bond and equity, the difference lies in whether or not we allow for restructuring in the event of default. We compare the prices of the two short-term bonds B_1 and B_1'' and of the two long-term bonds B_2 and B_2'', respectively. All the results we have obtained are proved analytically. For simplicity we make use of the graphs in order to explain the intuition of the results. The choice of parameters is arbitrary, the only restrictions of the parameters are those that guarantee that renegotiation takes place in case of default.

When deriving the equilibrium of the game we obtained that Bondholder 1 is better off undertaking restructuring independently of the action of Bondholder 2. Since in equilibrium Bondholder 1 chooses to restructure, it is obvious that his payoff has to be higher when restructuring takes place than when liquidation occurs. Moreover, by restructuring he postpones or avoids costly liquidation giving the firm the possibility to recover. As a result, we find that his expected payments are higher and consequently, that the price at date 0 of the short-term bond is higher if strategic interaction between bondholders is allowed.

Proposition 3. *The price at date 0 of the short-term bond is higher if renegotiation is allowed, that is*

$$P_1(D_1, D_2, V_0) \geq \tilde{P}_1(D_1, D_2, V_0),$$

where $P_1(D_1, D_2, V_0)$ and $\tilde{P}_1(D_1, D_2, V_0)$ are defined in Lemma A.1 and A.2 in Appendix A.

In Fig. 2 we see that there are two ranges for V_0 where the price of the short-term bond is higher where we allow for debt restructuring. The values of V_0 for which this happens are the two possible cases when the value of the firm is lower than D_1 but higher than $\overline{V}$. Note that the increase in price is significant. In the first case when the value of the firm is lower than D_1 the increase might be very high in both the upper and lower state). This big increase is due to the fact that here the value of the short-term bond without restructuring was very small. However, even where the value of the firm is higher (and default is possible only in the lower state) it still increases significantly. In our example, the maximum increase is almost 10%, while for other values of parameters it rises to 15% (see Table 1). Since the price of the short-term bond is higher, we have a decrease in the spread of short-term bond when strategic behavior is allowed. For our example this decrease varies from 19% to 22%. Note that as the value of the long-term debt D_2 increases, the range where we have the difference between the prices of the short-term bonds shrinks.

Proposition 4. *The price at date 0 of the long-term bond is higher if renegotiation is permitted, that is*

$$P_2(D_1, D_2, V_0) \geq \tilde{P}_2(D_1, D_2, V_0),$$

where $P_2(D_1, D_2, V_0)$ and $\tilde{P}_2(D_1, D_2, V_0)$ are defined in Lemma A.1 and A.2 in Appendix A.

However, the equilibrium payoff of Bondholder 2 is lower than where both bondholders liquidate. Since the Bondholder 1's best response is to restructure when Bondholder 2 liquidates, liquidation by both bondholders will not be an equilibrium for $V_1 > \overline{V}$. So, Bondholder 2 ends up with a payoff lower than is where we do not allow for debt restructuring. Since where we allow for

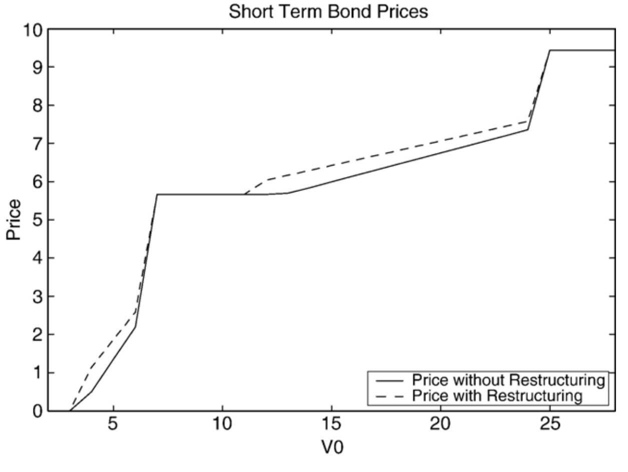

Fig. 2. Comparison of the short-term prices. The values of parameters are: $D_1 = 10$, $D_2 = 6$, $K = 0.4$, $L = 0.02$, $p = 0.6$, $u = 1.5$, $d = 0.4$.

Table 1
Short-term bond price increase (in percentages)

V_0	D_2						
	4	5	6	7	8	9	10
9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	5.92%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
11	6.58%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
12	6.03%	6.68%	0.00%	0.00%	0.00%	0.00%	0.00%
13	5.50%	8.34%	0.00%	0.00%	0.00%	0.00%	0.00%
14	5.00%	7.74%	0.00%	0.00%	0.00%	0.00%	0.00%
15	4.52%	7.16%	9.70%	0.00%	0.00%	0.00%	0.00%
16	4.06%	6.62%	9.48%	0.00%	0.00%	0.00%	0.00%
17	3.63%	6.10%	8.86%	10.45%	0.00%	0.00%	0.00%
18	3.21%	5.60%	8.27%	11.26%	0.00%	0.00%	0.00%
19	2.82%	5.13%	7.70%	10.59%	11.21%	0.00%	0.00%
20	2.44%	4.67%	7.16%	9.94%	13.07%	0.00%	0.00%
21	2.07%	4.24%	6.65%	9.33%	12.34%	0.00%	0.00%
22	1.72%	3.83%	6.16%	8.75%	11.65%	14.23%	0.00%
23	1.38%	3.43%	5.69%	8.20%	10.99%	14.14%	0.00%
24	1.06%	3.05%	5.24%	7.67%	10.37%	13.39%	14.98%
25	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

The parameters value are: $D_1=10$, $p=0.6$, $u=1.5$, $d=0.4$, $L=.4$, $K=.02$.

restructuring the expected payoffs are smaller, we also find that the price at date 0 of the long-term bond is smaller and consequently, the spread is higher. Similarly to the case of short-term bond prices, we have two regions where prices differ (see Fig. 3). Note that the decrease in the price of the long-term debt is smaller than the increase in the price of the short-term debt. Thus in our example the range is from 6.8% to 10% (see Table 2). Also note here that as the value of the short-term debt D_1 increases, the range where we have the difference between the prices of the long-term bonds expands.

As we have already explained, the bondholders' payoffs are significantly changed when we allow for debt restructuring. However, this is not the only issue here. We are interested in seeing

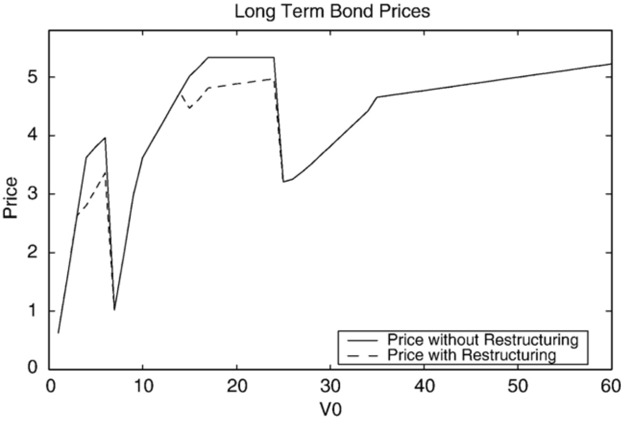

Fig. 3. Comparison of the long-term prices. The values of parameters are: $D_1=10$, $D_2=6$, $K=0.4$, $L=0.02$, $p=0.6$, $u=1.5$, $d=0.4$.

Table 2
Long-term bond price decrease (in percentages)

V_0	D_1								
	7	8	9	10	11	12	13	14	15
14	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
15	10.31%	10.56%	10.74%	10.92%	11.11%	11.30%	11.51%	11.72%	11.94%
16	10.26%	10.26%	10.26%	10.60%	10.78%	10.96%	11.15%	11.35%	11.56%
17	9.83%	9.83%	9.83%	9.83%	10.08%	10.24%	10.42%	10.60%	10.78%
18	0.00%	9.41%	9.41%	9.41%	9.41%	9.41%	9.72%	9.88%	10.05%
19	0.00%	8.98%	8.98%	8.98%	8.98%	8.98%	8.98%	9.20%	9.36%
20	0.00%	0.00%	8.55%	8.55%	8.55%	8.55%	8.55%	8.55%	8.55%
21	0.00%	0.00%	8.13%	8.13%	8.13%	8.13%	8.13%	8.13%	8.13%
22	0.00%	0.00%	7.70%	7.70%	7.70%	7.70%	7.70%	7.70%	7.70%
23	0.00%	0.00%	0.00%	7.27%	7.27%	7.27%	7.27%	7.27%	7.27%
24	0.00%	0.00%	0.00%	6.85%	6.85%	6.85%	6.85%	6.85%	6.85%
25	0.00%	0.00%	0.00%	0.00%	6.42%	6.42%	6.42%	6.42%	6.42%
26	0.00%	0.00%	0.00%	0.00%	5.99%	5.99%	5.99%	5.99%	5.99%
27	0.00%	0.00%	0.00%	0.00%	5.57%	5.57%	5.57%	5.57%	5.57%
28	0.00%	0.00%	0.00%	0.00%	0.00%	5.14%	5.14%	5.14%	5.14%
29	0.00%	0.00%	0.00%	0.00%	0.00%	4.71%	4.71%	4.71%	4.71%
30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.29%	4.29%	4.29%
31	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.86%	3.86%	3.86%
32	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.43%	3.43%	3.43%
33	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.01%	3.01%

The parameters value are: D_2=6, p=0.6, u=1.5, d=0.4, L=.4, K=.02.

how the presence of strategic behavior is reflected in the payments, and consequently, in the prices of bonds, but also to understand what lies behind these changes. Thus, there are different channels in which strategic behavior comes into play: through the change in the value of the firm, through the reallocation of payments, through the possible changes in the hierarchy of debt or avoidance of costly liquidation.

We should also emphasize that in our models the bankruptcy and restructuring costs are anticipated by the bondholders and therefore, they are incorporated in prices. The same happens with the bankruptcy procedure. Where the bondholders do not restructure the debt, the bankruptcy code predicts liquidation similar to Chapter 7 of U.S. Bankruptcy Code. Our model suggests thus, that the prices of the bonds are also affected by the bankruptcy procedure.

Consider finally, the short-term bonds B_1'' and B_1'. As we have already explained, the two bonds have the same face value D_1 and maturity date 1. Their difference lies in the fact that they are outstanding bonds in firms with different capital structures. We compare the two prices and we find that the short-term bond has a higher price when this is the only outstanding bond (see Fig. 4). The result is very intuitive. Since in the event of default payments are made according to the priority rule, the price of a junior bond is influenced by the presence of another, senior bond. Where default occurs, the owner of the bond B_1' is paid immediately after the liquidation costs are paid, while the owner of the bond B_1'' also has to wait for the senior debt to be paid. So, the price of the bond B_1'' with maturity t=1 is strictly lower in the presence of another senior debt (even if this senior debt has later maturity).

We also compared the price of the long-term bond B_2'' with the price of the bond B_2'. It is quite intuitive that the value of a senior debt is lower or equal in the presence of a junior debt with earlier maturity because the payment made to Bondholder 1 at date 1 decreases the value of the

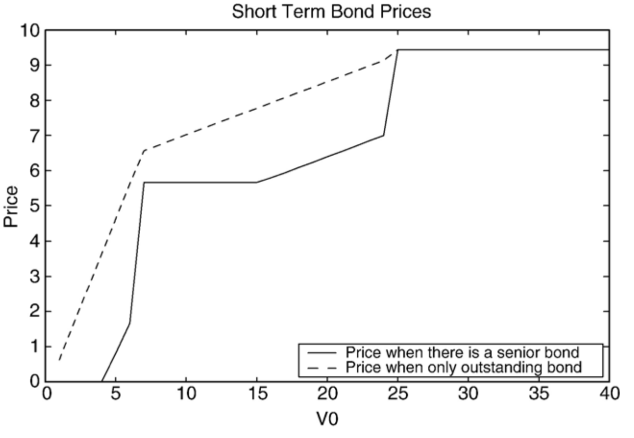

Fig. 4. Comparison of the short-term prices in two firms with different capital structure. The values of parameters are: $D_1=10$, $D_2=6$, $K=0.4$, $L=0.02$, $p=0.6$, $u=1.5$, $d=0.4$.

firm, and therefore, may decrease the payment to Bondholder 2 at date 2. However, this does not always seem to be the case. If at date 1, we have default on the obligation D_1, we have liquidation and Bondholder 2 receives $\min\left\{V_1-L, \frac{D_2}{1+r}\right\}$. Assume that the value of the firm is high enough, such that $\min\left\{V_1-L, \frac{D_2}{1+r}\right\} = \frac{D_2}{1+r}$. Consider now what happens with the bond B_2'. If the value of the firm is low enough to give rise to default of the second firm at date 2, the payment to B_2' will be lower than $\frac{D_2}{1+r}$ and therefore, the price of the bond B_2'' will be higher than the price B_2'. The insight is simple and it is the consequence of the fact that bond B_2'' is a senior bond. If the value of the firm is low, so it will almost certainly lead to default on B_2' in several states at date 2, Bondholder 2 might welcome liquidation at date 1 which leaves him better off. However, he can be better off by cashing in payment at date 1, only if the value of the firm is not too low. The two regions in Fig. 5 where the price of the senior bond is higher when there is a short-term bond correspond exactly to this case.

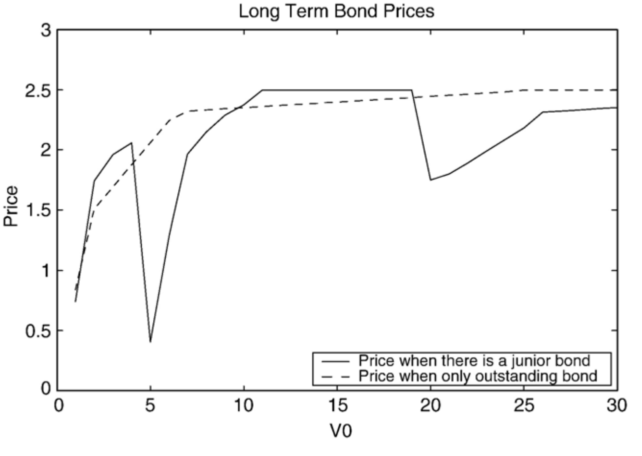

Fig. 5. Comparison of the long-term prices in two firms with different capital structure. The values of parameters are: $D_1=10$, $D_2=6$, $K=0.4$, $L=0.02$, $p=0.6$, $u=1.5$, $d=0.4$.

5. Conclusions

In this paper we attempt to derive the prices of debt and equity and to analyze the implications of strategic behavior and capital structure of a firm on the prices of bonds. Our main result is that both strategic behavior and the capital structure of the firm have important effects on the prices of bonds. To study these implications, we set our problem against three backdrops and we compared the prices we obtain. The whole analysis focussed on the determination of the lower reorganization boundary and on the payoffs received by agents. We first investigated the inference of strategic behavior of the agents on the prices of the bonds. To do so we compared the prices of the short-term and long-term bond in two firms with a similar capital structure. The only difference between the two firms lay in the behavior of the agents in case of default at date 1, in one setting allowing the agents to step in and restructure the debt. The model allowed us also to understand the importance of the covenants of the bonds where there are multiple creditors. We conclude that allowing for strategic behavior of bondholders leads to significant changes in bond prices. In addition, we found an increase in the value of the firm, but this increase takes place only when liquidation and restructuring cost are not zero. We conclude therefore, that strategic behavior partially offsets the loss produced by these costs. However, the strategic behavior by itself does not lead to any detriment in the value of the firm because it just reallocates the present funds. Thus, when there are no liquidation and restructuring costs, we find that the Modigliani–Miller theorem holds.

Second, we considered the effect of capital structure of the firm on the prices of bonds. We compared the prices of the short and long-term bond in the previous firm (without strategic behavior) with the prices of a short and a long-term bond, in a firm where the short-term bond and respectively the long-term bond are the only outstanding debt. It is quite intuitive that the presence of a senior bond decreases the value of a junior bond as compared with the case where the junior bond is the only bond outstanding. However, we also found that the presence of a junior bond with earlier maturity can decrease the price of a senior bond with later maturity. There are two cases where this happens. The first one is where the value of the firm at date 1 is small and the firm cannot meet its debt obligations. The firm defaults and goes bankrupt. Since bankruptcy involves significant costs, the payments due to the senior bond are also endangered. The second case takes place when the firm does not default at date 1, but the value of the firm is fairly low. At date 1, the firm pays the untitled debt D_1 for which it has to liquidate a part of its assets. Hence, the value of the firm V_1 decreases by D_1 and this induces a higher likelihood of default at date 2 on the senior bond.

Though simple in essence, our model suggests that the presence of multiple creditors and of a richer capital structure is an important issue to be considered in pricing corporate debt.

Acknowledgement

I am very grateful to Jordi Caballé for his helpful comments and kind guidance. I would like to thank also Ron Anderson, Giacinta Cestone, David Pérez-Castrillo, an anonymous referee and seminar participants at Bank of England, ESADE Business School, Financial Markets Group/LSE, Haskayne School of Business, HEC Montreal, IDEA Microeconomics Workshop, IESE Business School, Said Business School, University College London, Universitat Pompeu Fabra, Credit 2002 Conference, Assessing the Risk of Corporate Default, 2002 European Economic Association Meeting, the 7th Meeting of Young Economists for their comments. All the remaining errors are my own responsibility.

Appendix A

Lemma A.1. *The price at date 0 of the zero coupon bond maturing at t = 1, when there exists a positive probability of default is*

$$
\begin{aligned}
P_1(D_1, D_2, V_0) = \frac{1}{1+r} E_0 \Bigg[& \max\left\{ V_1 - L - \frac{D_2}{1+r}, 0 \right\} I_{\{V_1 | V_1 \leq K + \frac{D_2}{pu+(1-p)d}\}}(V_1) + \frac{1}{1+r} E_1 \\
& \times [\max\{0, \min\{V_2^* - D_2, D_1'\} - L \cdot I_{\{V_2^* | D_2 < V_2^* < D_2 + D_1'\}}(V_2^*)\}] \\
& \times I_{\{V_1 | K + \frac{D_2}{pu+(1-p)d} \leq V_1 < D_1\}}(V_1) + D_1 I_{\{V_1 | D_1 < V_1\}}(V_1) \Bigg]. \qquad (1)
\end{aligned}
$$

The price at date 0 of the zero coupon bond maturing at t = 2, is

$$
\begin{aligned}
P_2(D_1, D_2, V_0) = & \frac{1}{1+r} E_0 [\min\left\{ V_1 - L, \frac{D_2}{1+r} \right\} \cdot I_{\left\{ V_1 | V_1 \leq K + \frac{D_2}{pu+(1-p)d} \right\}}(V_1) \\
& + \frac{1}{1+r} E_1 [\min\{V_2^* - L, D_2\}] \cdot I_{\left\{ V_1 | K + \frac{D_2}{pu+(1-p)d} \leq V_1 < D_1 \right\}}(V_1) \\
& + \frac{1}{1+r} E_1 \left[\min\{\hat{V}_2, D_2\} - L \cdot I_{\{\hat{V}_2 | \hat{V}_2 < D_2\}}(\hat{V}_2) \right] \cdot I_{\{V_1 | D_1 \leq V_1\}}(V_1)]. \qquad (2)
\end{aligned}
$$

Lemma A.2. *The price at date 0 of the zero coupon bond maturing at t = 1, when there is a positive probability of default, is given by*

$$
\tilde{P}_1(D_1, D_2, V_0) = \frac{1}{1+r} E_0 \left[\max\left\{ V_1 - L - \frac{D_2}{1+r}, 0 \right\} \cdot I_{\{V_1 | V_1 \leq D_1\}}(V_1) + D_1 \cdot I_{\{V_1 | D_1 < V_1\}}(V_1) \right]. \qquad (3)
$$

The price at date 0 of the zero coupon bond maturing at t = 2, is

$$
\begin{aligned}
\tilde{P}_2(D_1, D_2, V_0) = & \frac{1}{1+r} E_0 [\min V_1 - L, \frac{D_2}{1+r} \cdot I_{V_1 | V_1 \leq D_1}(V_1) \\
& + \frac{1}{1+r} E_1 [\min\{\hat{V}_2, D_2\} - L \cdot I_{\{\hat{V}_2 | \hat{V}_2 < D_2\}}(\hat{V}_2)] \cdot I_{\{V_1 | D_1 < V_1\}}(V_1)]. \qquad (4)
\end{aligned}
$$

Sketch of proof Proposition 3. We will compare now the prices of the two short-term bonds B_1 and B_1'' using the formulas (1) and (3). The prices of the two bonds will be different for the values of the parameters where restructuring takes place. Since in equilibrium the bondholders choose to restructure and not to liquidate, it is clear that the payoffs following restructuring are higher than the ones following liquidation. Consequently, the payoffs to debtholders and therefore, the prices of the bonds, are higher when we allow for renegotiation then when we do not. □

Sketch of proof Proposition 4. The proof of this Proposition is very similar to the previous one. We use again Lemma A.1 and A.2 to define the prices for the two long-term bonds B_2 and B_2''.

The price of B_2 is given by (2) and the price of B_2'' is given by (4). We compute again the expectations and we compare the two values for different values of parameters taking into account that in equilibrium the payoffs in case of restructuring are higher then in case of liquidation. □

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